Clock Bias in GPS Systems

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1. Introduction

Determining true geometric ranges in GPS systems center around error analysis. Range biases are both atmospheric and physical; each independently effecting absolute range. The error budget for the carrier phase ranging is as follows:

$$\varphi = \rho + d_{\rho} + c(dt - dT) + \lambda N - d_{ion} + d_{drop} + \varepsilon_{m\varphi} + \varepsilon_{\varphi}$$

where, ρ is the true geometric range, d_{ρ} is orbital bias, c is the speed of light in a vacuum, dt is satellite clock bias, dT is receiver clock bias, λN is the integer number of wavelengths, and the remaining four variables are errors due to ionospheric and tropospheric delay, multipath and receiver noise, respectively. Note that the pseudo-range equation is similar yet disregards the λN expression. We will focus on clock biases as they account for a large portion of the overall error.

2. GPS Basics

First let's review the basics of GPS systems. Unlike EDM (Electronic Distance Measurement) methods, GPS is a one-range system. It is also referred to as a *passive* system as data is sent from the satellite to the receiver. Information cannot be sent back to the satellite from the receiver.

There are three main components of the system; GPS satellites, receivers and the ground control segment. The GPS constellation is comprised of 24 operational satellites, orbiting the earth at an altitude of 2,000 km and at a speed of approximately 17,000 miles per hour. A GPS receiver is used to determine where (and when) the satellite is in the sky, among other information. In space navigation, the astronomical position is known as the *ephemeris.*

Lastly, the ground control segment (GCS) consists of 12 monitoring stations worldwide, a master control station (MCS) located in Colorado Springs as well as broadcast antennae. The GCS plays a vital role in GPS systems, specifically in error management. Monitoring stations are equipped with GPS receivers and atomic clock standards. These stations gather GPS satellite data (the ephemeris and clock errors among other parameters) which is then fed into the MCS. Here, data are processed and compiled into the broadcast corrections which is sent on the S-band via broadcast antenna to individual satellites. This data update usually occurs at least once in a 24 hour period.

2.1 Ranging in GPS

As GPS is a one-range system, determining true geometric range requires the use of two independent clocks; one on board the orbiting satellite and another within the GPS user receiver. In simplistic terms, the range (distance), ρ, between a satellite and a receiver is function of the speed of light, c, an electromagnetic frequency represented as a sine wave and elapsed time, Δt.

$$\rho = c\,\Delta t$$

There are three frequencies used in GPS signals, L1, L2 and now L3. All bands live within the microwave region of the spectrum. When a signal is first emitted from the satellite, a replicated signal is also generated within the GPS receiver. The number of full rotations, N, as well as fractional parts are considered when determining geometric range. The comparison of the replicated signal and the satellite signal is the mechanism behind understanding partial rotations.

3. Clock Varieties

Most time-keeping devices involve the concept of isolation or rotation. For example, the old-fashioned pendulum clock used harmonic oscillations to track time. One full swing defines one period and the duration of time is determined by the length of the pendulum and gravitation pull.

3.1 Quartz-Based Clocks

Equipped in most land-based GPS receivers, is a clock powered by a piezoelectric quartz crystal (the same mineral found in most modern wrist watches). Also, there is a large mechanical component, as gears are used to track time. An oven-controlled crystal disk (sometimes referred to as OCXO [book]) is “squeezed” (via internal battery), generating a slight electric current and therefore a vibration. These oscillations are the mechanism to track time. The quartz crystal oscillates exactly 32,768 times per second.

This variety of clock is appealing because they are compact and relatively inexpensive as they have long life-spans and low power requirements. However, quartz crystal clocks found in GPS receivers are influenced by external factors such as temperature, shock and vibrations. They also have sub-par accuracy ratings and can drift up to 0.1 ns per second.

3.2 Atomic Clocks

The use of atomic clocks in satellites pushed GPS into the modern era during the mid-20th century. When an atom changes energy states (specifically from low-high), it emits an electromagnetic pulse in a discrete, constant, frequency, known as the resonant frequency. Atoms of the same type will share the same resonant frequency, making atomic clocks extremely precise in time keeping. The first atomic clock was successfully built in the 1940s by a physics professor at Columbia University, Isidor Rabi. (Although the idea was first proposed by Lord Kelvin in 1879).

The only stable isotope of Cesium, Cesium-133, was used in the first atomic clock described above. It has a resonant frequency of exactly 9,192,631,770 cycles per second. It gives life to the famous NIST-F1 clock currently held in Boulder, Colorado. Of course this frequency is different from the other common atom used in atomic clocks, Rubidium-87. Both types give roughly the same accuracy, but the Rubidium clock is much less expensive and more compact. There exists a third type of atomic clock, using Hydrogen, but its reliability digresses over time. In contrast, the NIST-F1 atomic clock will be accurate within one second for the next 100 million years.

4. Time Varieties: UTC and GPS

Before we consider clock errors, we first must understand the different time standards used in GPS. *Universal Time Coordinated* (UTC) is a standard developed in 1884 which served as the backbone for world time zones. There are two components that determine UTC. First, hundreds of atomic clocks define an International Atomic Time. Secondly, UTC also considers astronomical time, which is based on the rotation of the earth. As atomic clocks are more stable than the earth’s rotation itself, leap seconds are periodically integrated to UTC in order for the pace provided by the IAT to coincide with the actual length of a day on earth.

Another time standard utilized is GPS time. While UTC is the standard for the civil world, GPS time is used in GPS systems. Although the *rate* of these two standards are essentially identical, the times themselves are not. This is due to the fact the GSP time does not incorporate leap seconds as solar time is irrelevant to GPS systems. January 6th, 1980 UTC and GSP time were in sync. As of 2012, UTC lagged GPS time by 16 seconds.

5. Clock Bias

Time plays a fundamental role in geographical positioning. A GPS satellite system requires the simultaneous usage of four satellites within the constellation. In order to determine a unique solution set, four equations must be equipped with the four unknowns (x,y,z directions and time).

5.1 Satellite clock Bias

Satellite clock bias is often one of the largest errors credited to the overall error budget. On board each satellite are atomic clocks. Therefore, clock bias, represented as dt, is unique and independent of other external clock errors. Consequently, time determined by satellite clocks vary by from GPS time (by up to 1 millisecond). Simply adjusting satellite clocks to reflect GPS time would greatly diminish the life-span of the atomic clock. We will touch upon alternative solutions to this time adjustment later.

5.1.1 Relativistic Effects

In the early 1900s, a German physics Albert Einstein, transformed the foundation of physics and astronomy when he introduced his theories of relatively. In the context of GPS, Special Relatively theory explains that atomic clocks on-board satellites will *appear* to operate slower than clocks positioned on earth, due to a much higher angular velocity. This component (velocity) of the time dilation is effected by a factor of

$$\frac{v^2}{2c^2}$$

where v is the satellites orbital velocity and c is the speed of light. Given the orbital speed of 3.874 km/s, GPS clocks are delayed by about 7 μs per day.

However, the theory of General Relativity acts to counter this slowing of GPS satellite clocks. The underlying theme in this theory states that gravity effects the speed in which clocks operate. As local clocks on the surface of the earth experience a higher gravitational pull than satellite clocks orbiting the earth some 2,000 km away, they *appear* to run slower. The gravitation time dilation results in satellites gaining about 46 μs per day compared to earth-bounded clocks. The addition of both time dilations equate to about 39μs per day. That is, GPS satellites gain roughly 39 μs over the course of a day compared to clocks on earth due to relativity effects. See Figure 1 below for details regarding the time dilation effects of GPS satellites.

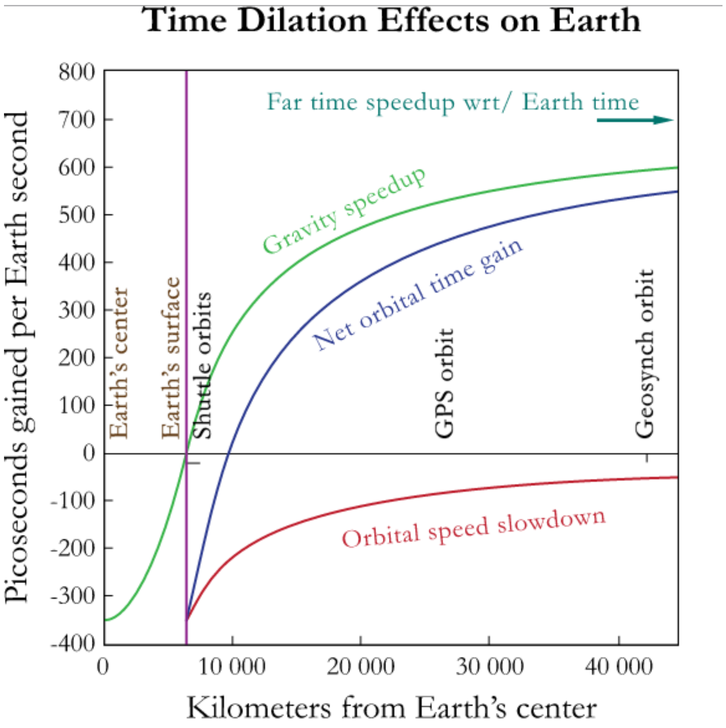

Figure 1: Time Dilation

In addition to general and special relatively effects, a concept known as the Sagnac effect also contributes to the desynchronization of clocks. Due to the rotation of the earth, receiver clocks are also in motion. At worse, this influence offsets the clock system by only about 133 ns (0.133 μs) per day. Luckily all three effects mentioned, special relativity, general relative and the Sagnac effect can all be precisely mathematically predicted, and therefore removed from the overall error budget.

5.1.2 Clock Drift

As mentioned previously, satellite clocks all run at slightly different speeds. That is, they tend to *drift* apart from not only one another but from standard GPS time. The maximum difference allowed is just 1 millisecond. The Master Control Station (MCS) located in Colorado Springs continuously receives data from orbiting satellites that is stored in 12 monitoring stations throughout the world. Once the MCS gathers and processes data, information (including clock errors) are then uploaded back to each individual satellite via broadcast clock corrections. Also, recall that clock errors are a component of the original Navigation (NAV) message sent from the satellite to the GPS receiver.

5.2 Receiver Clock Bias

Errors due to ground-based GPS receivers are much smaller than those found in orbiting satellites. Recall that quartz-crystal clocks are not nearly as accurate as atomic clocks. The replicated signal generated in receivers are directly associated with the stability of the oscillator. Luckily this bias is minimal. We will now consider standard solutions to clock biases.

6. Solutions

We have already mentioned a few techniques used to alleviate clock bias. Relativistic effects applied to satellite clocks can essentially be mathematically removed from the clock error budget. Also, the broadcast clock correction managed by the control segment helps update clock biases back to the GPS satellite. There are many solutions to GPS biases, but one method in particular pertains to clock errors, specifically, differencing. It involves the simultaneous usage of at least two observations. There are four main varieties of differencing. Also, we will discuss the role that the ground control segment plays in clock error management.

6.1 Single Differencing Between Receivers

A process known as single differencing is possible when exactly two receivers are observing a particular satellite at the same time. When baselines (the distance between receivers) are small, error bounds are even less as atmospheric, orbital and ephemeris errors are nearly identical. This allows us to analysis satellite clock bias, dt, almost exclusively by comparing data gathered from each receiver as other bias will the essentially the same. In fact, position can be calculated within a margin of centimeters when baselines are less 10 km and within meters when within a few hundred kilometers. Satellite clock errors, as orbital and atmospheric, are deemed negligible when baselines are relatively small.

6.2 Single Differencing Between Satellites

Similarly, we can compare data gathered from a single receiver, transmitted simultaneously from two different satellites. As only one ground-based clock is being utilized, receiver clock errors are greatly reduced, and are even considered eliminated. Like single differencing between receivers, atmospheric conditions are nearly identical, therefore, ionospheric and tropospheric biases are better managed. In contrast, as each satellite orbital bias will be different, single differencing between satellites will not necessarily improve actual positioning knowledge of the satellites.

6.3 Double and Triple Differencing

The combination of each type of single differencing is referred to as double differencing. As single differencing between receivers essentially eliminates satellite clock bias and single differences between satellites eliminates receiver clock bias, double differencing, for all intents and purposes, removes all clock biases from the error budget. The only condition is that baselines are relatively small. Furthermore, triple differencing is the combination of two simultaneous double differencing over difference epochs. However, this variety of differencing provides no additional improvement of clock bias compared to double differencing.

6.4 Ground Control Segment

Until 2005, there were only six monitoring stations feeding data to the MCS. This configuration allowed for a satellite to go monitored for up to two hours. As GPS requires the simultaneous usage of 4 satellites, this scenario was far less than ideal. During the Fall of '05, the Legacy Accuracy Improvement Initiative added six additional NGS stations worldwide; doubling the number of monitoring stations. Currently, there are at least 3 monitoring stations tracking every satellite at any given time.

The additional stations provide the MCS with a constant flow of data, include satellite clock data. One most important consequence of such a vast network of monitoring stations is that it provides the

MCS with *geographical diversity*. Consequently, the analysis of all error is much more thorough as more data being collected. With more stations, the MCS is able to process errors with better reliability and consistency. Better *error analysis* results in heighten accuracy for the overall GPS system.

6.5 Conclusions

In summary, both receiver and satellite clock biases contribute the overall error budget. Relativity effects can be mathematically predicted, and therefore removed from system. Individual satellite clock drift is not only accounted for in the original NAV message, but is also considered in the clock correction broadcast, processed and uploaded by the MCS. The concept of double differencing in GPS system is critical as this technique acts to eliminate both satellite and receiver clock biases. Lastly, we discussed the importance of the abundance as well as the geographical variety in monitoring stations as this greatly improves the overall accuracy of GPS.

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